Three Spin Spiky Strings in β-deformed Background

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ABSTRACT: We study rigidly rotating strings in β-deformed $AdS_5 \times S^5$ background with one spin along AdS_5 and two angular momenta along S^5. We find the spiky string solutions and present the dispersion relation among various charges in this background. We further generalize the result to the case of four angular momenta along $AdS_5 \times S^5_\gamma$.

KEYWORDS: AdS-CFT Correspondence, Bosonic Strings.

Contents

1. Introduction

AdS/CFT correspondence [1] relates the spectrum of free strings on $\mathrm{AdS}_5\times \mathrm{S}^5$ with the spectrum of operator dimensions in the $N = 4$ Supersymmetric Yang-Mills (SYM) theory in four dimensions. This mapping is highly nontrivial and challenging because of our lack of understanding of the full string theory spectrum. Hence, it is instructive to look at both the gauge and gravity theories at certain limits such as large angular momentum limit and then compare the spectrum on both sides. Further, $N = 4$ SYM theory can be described by integrable spin chain model where the anomalous dimension of the gauge invariant operators were found [2]. It was further noticed that string theory has as integrable structure in the semiclassical limit and the anomalous dimension in the N = 4 SYM can be derived from the relation between conserved charges of the worldsheet solitonic string solution of the dual string theory on $\mathrm{AdS}_5\times \mathrm{S}^5$ background.

In the past few years the integrability of both string theory and gauge theory side has played a key role in proving the duality conjecture better. The most frequently studied cases were rotating and pulsating strings solutions in certain limits of spin waves in long-wave approximation and interesting observations were made in [3], [4], [5], [6], [7], [8]. Another interesting case is the low lying spin states which are equivalent to the so called magnon states. In this connection, Hofman and Maldacena in [9] have derived a mapping between particular state (magnon) of spin chain with the semiclassical string states on $R_t \times S^2$. Further it was realized that magnons are special cases of more general solutions known as spikes and the dual gauge theory operators have been analyzed in detail in [10]. It was also observed in [11] that both giant magnon and single spike solutions can be viewed as two different limits of the same rigidly rotating strings on S^2 and S^3. In this connection, a large class of multispin spiky string and giant magnon solutions have been studied, in

various backgrounds including the orbifolded and non-AdS backgrounds, for example in [12]

We are interested in studying a class of spiky string solution in the so called Lunin-Maldacena background [13]. The integrability of Lunin-Maldacena background has been studied in [14], [15], [16]. The giant magnon and single spike solutions are studied in detail including the integrable models, for example in [17], [18].

More recently in [19], [20], [21], more general class of solutions with three divergent angular momenta have been studied and interesting dispersion relations among various conserved charges have been obtained. In the present paper, we wish to generalize the result of [22] in a β-deformed background with one(two) spin in AdS and two spins on the deformed sphere. Knowing such solutions on the gravity side will definitely help us in finding out the nature of corresponding operators on the gauge theory side.

The rest of the paper is organized as follows. In section-2 we write the relevant part of the Lunin-Maldacena background which will be useful for studying the rigidly rotating string on this background. In section 3, we study the rotating open string in $AdS_3 \times S^3_\gamma$ backgrounds with one spin along the AdS_3 and two angular momenta along the deformed sphere. We compute all the conserved charges and find two limiting cases corresponding to giant magnon and single spike solutions. We write down the dispersion relation among various divergent momenta in both cases. We further generalize the above solutions to the case of rotating string with two spins along the AdS_3 and two angular momenta along the deformed sphere. We write down the corresponding dispersion relation for the giant magnon solution. Finally in section 4, we conclude with some remarks.

2. β-deformed $AdS_5 \times S^5$ background

Here we present the general background for β-deformed $AdS_5 \times S^5$ found by Lunin and Maldacena[13]. This background is obtained from pure $AdS_5 \times S^5$ by a series of STsTS transformations [23], which is dual to the Leigh-Strassler marginal deformations of $N = 4$ SYM [24]. The deformed parameter $\beta = \gamma + i\sigma_d$ in general is a complex number, but here we restrict β to its real part only. Thus, the relevant metric component of the supergravity background dual to real β-deformations of $N = 4$ SYM is:

$$ds^2 = R^2 \left(ds^2_{AdS_5} + \sum_{i=1}^{3} (d\mu_i{}^2 + G\mu_i{}^2 d\phi_i^2) + \tilde{\gamma}^2 G \mu_1^2 \mu_2^2 \mu_3^2 (\sum_{i=1}^{3} d\phi_i^2 \right), \qquad (2.1)$$

which also have the dilaton, Ramond-Ramond (RR) and Neveu-Schwarz-Neveu-Schwarz (NS-NS) fields.

The antisymmetric form of the B-field relevant for our classical string analysis is:

$$B = R^2 \tilde{\gamma} G (\mu_1^2 \mu_2^2 d\phi_1 d\phi_2 + \mu_2^2 \mu_3^2 d\phi_2 d\phi_3 + \mu_1^2 \mu_3^2 d\phi_1 d\phi_3), \qquad (2.2)$$

where

$$\tilde{\gamma} = R^2 \gamma, \quad R^2 = \sqrt{4\pi g_s N},$$

$$G = \frac{1}{1+\tilde{\gamma}^2(\mu_1^2\mu_2^2+\mu_2^2\mu_3^2+\mu_1^2\mu_3^2)},$$

$$\mu_1 = \sin\theta\cos\psi, \quad \mu_2 = \cos\theta, \quad \mu_3 = \sin\theta\sin\psi. \tag{2.3}$$

3. Semiclassical Strings on $\mathbf{AdS}_3\times\ \mathbf{S}^3_\gamma$

We restrict the motion of the string on $AdS_3\times S^3_\gamma \subset AdS_5\times S^5_\gamma$. This space can be achieved by putting $\mu_3 = 0$, $\phi_3 = 0$ i.e, $\psi = 0$, $\phi_3 = 0$ in (2.1) - (2.3). Thus the metric components of the deformed $AdS_3\times S^3_\gamma$ background is:

$$ds^2 = -\cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho d\phi^2 + d\theta^2 + G\sin^2\theta d\phi_1^2 + G\cos^2\theta d\phi_2^2, \tag{3.1}$$

where $G = \frac{1}{1+\tilde{\gamma}^2\sin^2\theta\cos^2\theta}$ and the relevant non-zero component of B-field due to the series of T-duality transformations is given by

$$B_{\phi_1\phi_2} = \tilde{\gamma}G\sin^2\theta\cos^2\theta. \tag{3.2}$$

The Polyakov action for the fundamental string in this background is given by

$$S = \frac{T}{2}\int d\tau d\sigma\Big[-\cosh^2\rho(\dot{t}^2-t'^2)+\dot{\rho}^2-\rho'^2+\sinh^2\rho(\dot{\phi}^2-\phi'^2)+\dot{\theta}^2-\theta'^2+$$
$$G\sin^2\theta(\dot{\phi_1}^2-\phi_1'^2)+G\cos^2\theta(\dot{\phi_2}^2-\phi_2'^2)+2\tilde{\gamma}G\sin^2\theta\cos^2\theta(\dot{\phi}_1\phi_2'-\phi_1'\dot{\phi}_2)\Big], \tag{3.3}$$

where the 'dot' and 'prime' denote the derivatives with respect to τ and σ respectively and $T = \frac{\sqrt{\lambda}}{2\pi}$, where λ is the 't Hooft coupling constant. We take the following anstaz for the rotating open string

$$t = \tau + F_1(y), \quad \rho = \rho(y), \quad \phi = \omega_1(\tau + F_2(y)),$$

$$\phi_1 = \tau + F_3(y), \quad \theta = \theta(y), \quad \phi_2 = \omega_2(\tau + F_4(y)), \tag{3.4}$$

where $y = a\sigma - b\tau$. Solving the equations of motion for t, ϕ, ϕ_1 and ϕ_2, we have the following expression for F_1, F_2, F_3 and F_4

$$F_{1y} = \frac{1}{a^2-b^2}\left(\frac{A_1}{\cosh^2\rho} - b\right),$$
$$F_{2y} = \frac{1}{a^2-b^2}\left(\frac{A_2}{\sinh^2\rho} - b\right),$$
$$F_{3y} = \frac{1}{a^2-b^2}\left(\frac{A_3}{G\sin^2\theta} - b - a\tilde{\gamma}\omega_2\cos^2\theta\right),$$

$$F_{4y} = \frac{1}{a^2-b^2}\left(\frac{A_4}{G\cos^2\theta} - b + \frac{a\tilde{\gamma}}{\omega_2}\sin^2\theta\right), \tag{3.5}$$

where $F_y = \frac{\partial F}{\partial y}$.

Now, the two virasoro constraints $T_{\tau\sigma} = 0$ and $T_{\tau\tau} + T_{\sigma\sigma} = 0$ give the following two equations

$$\begin{aligned}
\rho_y^2 + \theta_y^2 &= \cosh^2\rho(-\frac{1}{b}F_{1y} + F_{1y}^2) - \omega_1^2\sinh^2\rho(-\frac{1}{b}F_{2y} + F_{2y}^2) \\
&\quad -G\sin^2\theta(-\frac{1}{b}F_{3y} + F_{3y}^2) - \omega_2^2 G\cos^2\theta(-\frac{1}{b}F_{4y} + F_{4y}^2) \\
\rho_y^2 + \theta_y^2 &= \cosh^2\rho(\frac{1}{a^2+b^2} - \frac{2b}{a^2+b^2}F_{1y} + F_{1y}^2) - \omega_1^2\sinh^2\rho(\frac{1}{a^2+b^2} - \frac{2b}{a^2+b^2}F_{2y} + F_{2y}^2) \\
&\quad -G\sin^2\theta(\frac{1}{a^2+b^2} - \frac{2b}{a^2+b^2}F_{3y} + F_{3y}^2) - \omega_2^2 G\cos^2\theta(\frac{1}{a^2+b^2} - \frac{2b}{a^2+b^2}F_{4y} + F_{4y}^2),
\end{aligned} \tag{3.6}$$

Using (3.5) in (3.6) we get the following relation among various constants

$$-A_1 + \omega_1^2 A_2 + A_3 + \omega_2^2 A_4 = 0 \tag{3.7}$$

Now, the equation of motion for ρ becomes

$$(a^2-b^2)\rho_{yy} + \frac{1}{a^2-b^2}\sinh\rho\cosh\rho\left(\frac{A_1^2}{\cosh^4\rho} - a^2 - \frac{\omega_1^2 A_2^2}{\sinh^4\rho} + \omega_1^2 a^2\right) = 0\ , \tag{3.8}$$

where $\rho_{yy} = \frac{\partial^2\rho}{\partial y^2}$. We can compute equation of motion for θ from the first Virasoro constraint (3.6) which is given by the relation

$$\begin{aligned}
\theta_y^2 &= -\rho_y^2 + \cosh^2\rho(-\frac{1}{b}F_{1y} + F_{1y}^2) - \omega_1^2\sinh^2\rho(-\frac{1}{b}F_{2y} + F_{2y}^2) \\
&\quad -G\sin^2\theta(-\frac{1}{b}F_{3y} + F_{3y}^2) - \omega_2^2 G\cos^2\theta(-\frac{1}{b}F_{4y} + F_{4y}^2)\ .
\end{aligned} \tag{3.9}$$

Once we get the form of ρ_y then we will be able to compute θ_y. The conserved charges are

$$\begin{aligned}
E &= T\int d\sigma\ \cosh^2\rho(1 - bF_{1y}), \\
S &= \omega_1 T\int d\sigma\ \sinh^2\rho(1 - bF_{2y}), \\
J_1 &= T\int d\sigma\ G\sin^2\theta(1 - bF_{3y}), \\
J_2 &= \omega_2 T\int d\sigma\ G\sin^2\theta(1 - bF_{4y}).
\end{aligned} \tag{3.10}$$

3.1 Giant Magnon Solution

For finding out the giant magnon solution, we choose the integation constants as $A_1 = b, A_2 = 0, A_3 = b$ and $A_4 = 0$. The solution of equation (3.8) becomes

$$\rho_y^2 = \frac{1}{(a^2-b^2)^2}\left(a^2(1-\omega_1^2) - \frac{b^2}{\cosh^2\rho}\right)\sinh^2\rho. \tag{3.11}$$

Using (3.11) and the above integration constants in (3.9), we get the following expression for θ_y

$$\theta_y^2 = \frac{1}{(a^2-b^2)^2}\left[a^2(1-\omega_2^2)\cos^2\theta + b^2 - \frac{b^2}{G\sin^2\theta} + 2ab\tilde{\gamma}\omega_2\cos^2\theta\right] . \tag{3.12}$$

Note that the above equation can be rewritten as

$$\theta_y = \frac{\Omega_0}{a^2-b^2}\cot\theta\sqrt{\sin^2\theta - \sin^2\theta_0} , \tag{3.13}$$

where, $\sin\theta_0 = \frac{b}{\Omega_0}$, and $\Omega_0 = \sqrt{a^2-(a\omega_2-b\tilde{\gamma})^2}$. Now the conserved charges (3.10) become

$$\begin{aligned} E &= \frac{T}{a^2-b^2}\int d\sigma\ (a^2\cosh^2\rho - b^2), \\ \frac{S}{\omega_1} &= \frac{T}{a^2-b^2}\int d\sigma\ a^2\sinh^2\rho, \\ J_1 &= \frac{T}{a^2-b^2}\int d\sigma\ (a^2\sin^2\theta - b^2), \\ \frac{J_2}{\omega_2} &= \frac{T}{a^2-b^2}\int d\sigma\ a^2(1-\frac{b\tilde{\gamma}}{a\omega_2})\cos^2\theta. \end{aligned} \tag{3.14}$$

It is clear from the above expressions that we have the following relation among various conserved charges

$$E - J_1 = \frac{S}{\omega_1} + \frac{J_2'}{\omega_2} , \tag{3.15}$$

where $J_2' = \frac{a\omega_2}{\sqrt{a^2-{\Omega_0}^2}}J_2$. As the conserved charges are divergent, we use the same regularization technique as in [22] to remove the divergent part of the conserved charges. Let us write

$$\frac{S}{\omega_1} = \frac{2Ta}{a^2-b^2}\int_\infty^0 d\rho\frac{\sinh^2\rho}{\rho_y} = \frac{2T}{\sqrt{1-\omega_1^2}}\int_1^\infty dz\frac{z}{\sqrt{z^2-z_0^2}} , \tag{3.16}$$

where $z = \cosh\rho$ and $z_0 = \cosh\rho_0 = \frac{b}{a\sqrt{1-\omega_1^2}}$. Subtracting the divergent part of the integral (3.16), we have the regulated value given as

$$\frac{S_{reg}}{\omega_1} = -\frac{\sqrt{\lambda}}{\pi}\sqrt{\frac{1-z_0^2}{1-\omega_1^2}} . \tag{3.17}$$

From the above expression, we find the following relation

$$\frac{S_{reg}}{\omega_1} = -\sqrt{S_{reg}^2 + \frac{\lambda}{\pi^2}(1 - z_0^2)}\ . \tag{3.18}$$

Further, the time difference between two end points of the open string is given by

$$\begin{aligned}\Delta t &= -\frac{2b}{a\sqrt{1-\omega_1^2}} \int_{-\infty}^{\infty} d\rho \frac{\tanh\rho}{\sqrt{\cosh^2\rho - \frac{b^2}{a^2(1-\omega_1^2)}}} \\ &= -2\tan^{-1}\frac{z_0}{\sqrt{1-z_0^2}}\ .\end{aligned} \tag{3.19}$$

Now the equation (3.18) can be written in the following form

$$\frac{S_{reg}}{\omega_1} = -\sqrt{S_{reg}^2 + \frac{\lambda}{\pi^2}\cos^2\frac{\Delta t}{2}}\ . \tag{3.20}$$

Further, the angle difference between two end points of the open string is given by

$$\frac{\Delta\phi_1}{2} = \int_{\theta_0}^{\frac{\pi}{2}} d\theta \frac{\tilde{\gamma}\sqrt{a^2-\Omega_0^2}}{\Omega_0}\cot\theta \frac{\sin^2\theta + \frac{b}{\tilde{\gamma}\sqrt{a^2-\Omega_0^2}}}{\sqrt{\sin^2\theta - \sin^2\theta_0}} = \frac{\pi}{2} - \theta_0 + \frac{\tilde{\gamma}\sqrt{a^2-\Omega_0^2}}{\Omega_0}\cos\theta_0. \tag{3.21}$$

Now, we find the giant magnon dispersion relation as

$$(E - J_1)_{reg} = -\sqrt{S_{reg}^2 + \frac{\lambda}{\pi^2}\cos^2\frac{\Delta t}{2}} + \sqrt{J_2^2 + \frac{\lambda}{\pi^2}\sin^2\frac{\Delta\phi_1}{2}}. \tag{3.22}$$

This expression matches with that of [22] even if we are dealing with β-deformed background and has implicit dependence on the deformation parameter $\tilde{\gamma}$ in the definition of $\Delta\phi_1$.

3.2 Single spike Solution

To obtain the single spike solution, we chose the integration constants as: $A_1 = \frac{a^2}{b} = A_3$ and $A_2 = 0 = A_4$. The solution of equation (3.8) now becomes

$$\rho_y^2 = \frac{1}{(a^2-b^2)^2}\left(a^2(1-\omega_1^2) - \frac{a^4}{b^2\cosh^2\rho}\right)\sinh^2\rho. \tag{3.23}$$

Using (3.23) and the above integration constants in (3.9), we have the follwing expression for θ_y:

$$\theta_y = \frac{a\Omega_1}{a^2-b^2}\cot\theta\sqrt{\sin^2\theta - \sin^2\theta_1}, \tag{3.24}$$

where, $\sin\theta_1 = \frac{a}{b\Omega_1}$, and $\Omega_1 = \sqrt{1-(\omega_2 - \frac{a\tilde{\gamma}}{b})^2}$. Thus the conserved charges (3.10) becomes:

$$E = \frac{T}{a^2-b^2}\int d\sigma\ a^2\sinh^2\rho,$$

$$
\begin{aligned}
\frac{S}{\omega_1} &= \frac{T}{a^2-b^2}\int d\sigma\ a^2\sinh^2\rho,\\
J_1 &= -\frac{T}{a^2-b^2}\int d\sigma\ a^2\cos^2\theta,\\
\frac{J_2}{\omega_2} &= \frac{T}{a^2-b^2}\int d\sigma\ a^2(1-\frac{a\tilde{\gamma}}{b\omega_2})\cos^2\theta.
\end{aligned}
\tag{3.25}
$$

From (3.25), we get the follwing relation between the conserved charges

$$
E-J_1=\frac{S}{\omega_1}+\frac{J_2''}{\omega_2}\ , \tag{3.26}
$$

where $J_2''=\frac{b\omega_2}{\sqrt{1-\Omega_1{}^2}}J_2$. For completeness we wish to compute J_1 and J_2 as

$$
\begin{aligned}
J_1 &= -\frac{2Ta}{a^2-b^2}\ \int_{\frac{\pi}{2}}^{\theta_1}\frac{d\theta}{\theta_y}\cos^2\theta=\frac{2T}{\Omega_1}\cos\theta_1\ ,\\
J_2 &= \frac{2Ta}{a^2-b^2}\sqrt{1-\Omega_1^2}\ \int_{\frac{\pi}{2}}^{\theta_1}\frac{d\theta}{\theta_y}\cos^2\theta=-\frac{2T}{\Omega_1}\sqrt{1-\Omega_1^2}\cos\theta_1\ .
\end{aligned}
\tag{3.27}
$$

From (3.27), we have the follwing relation between J_1 and J_2

$$
J_1=\sqrt{J_2^2+\frac{\lambda}{\pi^2}\cos^2\theta_1}\ . \tag{3.28}
$$

This expression has already been found in [18]. One can also see that from (3.25) we have the following relation between E and S,

$$
E-\frac{S}{\omega_1}=0. \tag{3.29}
$$

3.3 Magnon solution with Four spins

In this section, we would like to generalize the results of the previous section to include two spins along AdS and two angular momenta along the deformed S^3_γ. The relevant metric and B-field is given by

$$
\begin{aligned}
ds^2 &= -\cosh^2\rho dt^2+d\rho^2+\sinh^2\rho(d\psi^2+\sin^2\psi d\xi_1^2+\cos^2\psi d\xi_2^2)\\
&\quad +d\theta^2+G\sin^2\theta d\phi_1^2+G\cos^2\theta d\phi_2^2\ ,\\
B_{\phi_1\phi_2} &= \tilde{\gamma}G\sin^2\theta\cos^2\theta\ ,\\
G^{-1} &= 1+\tilde{\gamma}^2\sin^2\theta\cos^2\theta\ .
\end{aligned}
\tag{3.30}
$$

We take the following anstaz

$$t = \tau + G_1(y), \quad \rho = \rho(y), \quad \psi = \text{constant}, \quad \xi_1 = \omega_1(\tau + G_2(y)),$$

$$\xi_2 = \omega_2(\tau + G_3(y)), \phi_1 = \tau + G_4(y), \quad \theta = \theta(y), \quad \phi_2 = \omega_3(\tau + G_5(y)), \tag{3.31}$$

where $y = a\sigma - b\tau$. Solving the equation of motion for the coordinates t, ξ_1, ξ_2, ϕ_1 and ϕ_2, we have the following expression for $G_1(y), G_2(y), G_3(y), G_4(y)$ and $G_5(y)$ respectively.

$$\begin{aligned}
G_1(y) &= \frac{1}{a^2 - b^2}\left(\frac{A_1}{\cosh^2\rho} - b\right), \\
G_2(y) &= \frac{1}{a^2 - b^2}\left(\frac{A_2}{\sinh^2\rho \sin^2\psi} - b\right), \\
G_3(y) &= \frac{1}{a^2 - b^2}\left(\frac{A_3}{\sinh^2\rho \cos^2\psi} - b\right), \\
G_4(y) &= \frac{1}{a^2 - b^2}\left(\frac{A_4}{G\sin^2\theta} - b - a\tilde{\gamma}\omega_3\cos^2\theta\right), \\
G_5(y) &= \frac{1}{a^2 - b^2}\left(\frac{A_5}{G\cos^2\theta} - b - \frac{a\tilde{\gamma}}{\omega_3}\cos^2\theta\right),
\end{aligned} \tag{3.32}$$

where A_1, A_2, A_3, A_4 and A_5 are integration constants, which satisfy the following relation, as derived from Virasoro constraints,

$$-A_1 + \omega_1^2 A_2 + \omega_2^2 A_3 + A_4 + \omega_3^2 A_5 = 0 \ . \tag{3.33}$$

The conserved charges derived from Virasoro constraints as explained earlier corresponding to t, ξ_1, ξ_2, ϕ_1 and ϕ_2 are E, S_1, S_2, J_1 and J_2 respectively. The charges are shown to satisfy the following dispersion relation among them

$$E - J_1 = \frac{S_1}{\omega_1} + \frac{S_2}{\omega_2} + \frac{\tilde{J}_2}{\omega_3} \ , \tag{3.34}$$

where $\tilde{J}_2 = \frac{J_2}{1 - \frac{b\tilde{\gamma}}{a\omega_3}}$. While deriving the above relation we have used $A_1 = b = A_4$, and $A_2 = 0 = A_3 = A_5$. One can further use the same kind of regularization technique as discussed in previous section to get a formal expression for the giant magnon dispersion relation in the presence of two spins along AdS_5 and two angular momenta along S^5. We skip the details here.

4. Conclusions

In this paper, we have found a class of giant magnon and single spike string solutions in less supersymmetric real β-deformed Lunin-Maldacena background with three spins along various directions of AdS_5 and S^5. The relation among conserved quantities in (3.15) is

similar to the undeformed $AdS_3 \times S^3$ giant magnon relation obtained in [22]. As expected, for zero deformed parameter i.e, $\tilde{\gamma} = 0$, we get the same value of J_2 and the same relation among the charges as derived in [22]. Thus, we get the result as expected in [17], where the authors claimed that the deformed parameter should not appear explicitly in the dispersion relation, however can be absorbed in the definition of the conserved charges. As argued in [17] if the magnon dispersion relation depends explicitly on the deformed parameter then in general we cannot find integrable spin chain systems. We also discarded the divergent terms in (3.22) and found the regularized dispersion relation which is superposition of two magnon bound states where the worldsheet momentum is shifted by a factor $2\pi\gamma$, as in [17],[18]. At this point, it is worth mentioning about [25], where a class of magnon solutions were derived and it was shown that in the limit of $J_2 \to \infty$, the dispersion relation was independent of the deformation parameter. In the present case however we would like to stress that our solutions are very similar to the ones presented in [18], because if we switch off the spin along the AdS space we get back dispersion relation presented there. However it will be interesting to generalize the solutions of [25] to include an extra spin along the AdS direction and check the finite size correction to the magnon and spike dispersion relation. It would also be interesting to look for the dual operators on the boundary, as one would expect from the AdS/CFT duality. The exact nature of the operators are unknown. But the expectations from [26] leads us to believe that such dual operators would exist. It would really be challenging to construct such operators dual to the spiky strings presented in this paper.

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